Planned Scheduling for Economic Power Sharing in a CHP-Based Micro-Grid

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***Abstract*—At the planning of combined heat and power (CHP)-based micro-grid, its distributed energy resources (DER) capacity is to be selected and deployed in such a way that it becomes economically self-sufficient to cater all the loads of the system without utility's participation. Economic deployment of DERs is meant to select optimal locations, optimal sizes, and optimal technologies. Optimal locations and sizes, which are independent of CHP-based DERs types, are selected, here, by loss sensitivity index (LSI) and by loss minimization using particle swarm optimization (PSO) method, respectively. In a micro-grid, both fuel costs and NO_x emissions are, mainly, dependent on types of DERs used. So the main focus of the present paper is to incorporate originality in ideas to evaluate how different optimal output sets of DER-mix, operating within their respective capacity limits, could share an electrical tracking demand, economically, among micro-turbines and diesel generators of various sizes, satisfying different heat demands, on the basis of multi-objective optimization compromising between fuel cost and emission in a 4-DER 14-bus radial micro-grid. Optimization is done using differential evolution (DE) technique under real power demand equality constraint, heat balance inequality constraint, and DER capacity limits constraint. DE results are compared with PSO.**

***Index Terms*—Diesel generator, differential evolution, economic emission load dispatch, loss sensitivity index, micro-turbine, particle swarm optimization.**

Nomenclature

DER	Distributed energy resources.
CHP	Combined heat and power.
Mt	Micro-turbine.
Dg	Diesel generator.
DG	Distributed generator/generation.
DE	Differential evolution.
P_L	System electric loss (kW).
PG_i	Active power generation of ith DER(kW).
$PG_{\mathrm{imax}}, PG_{\mathrm{imin}}$	Upper and lower limits of $PG_i(kW)$, respectively.
P_i	Active power injection at ith bus (kW).
δ_i	Phase angle of ith bus.
U_i	Voltage of ith bus (p.u.).
$U_{\mathrm{imax}}, U_{\mathrm{imin}}$	Upper and lower bounds of U_i (p.u.), respectively.
QG_i	Reactive power generation of ith DER.
$QG_{\mathrm{imax}}, QG_{\mathrm{imin}}$	Upper and lower bounds of QG_i (kvar), respectively.
P_D	Total electric demand (kW).
S_{ij}	Line flow from ith to jth bus (kW).
N	Total numbers of DERs.
C	Total cost of emission plus fuel ($/h).
$f_1, f_{1\,\max}$	Fuel cost function and its value at PG_{imax}, respectively ($/h).
$f_2, f_{2\,\max}$	NO_{x} emission function and its value at PG_{imax}, respectively (g/kWh).
W	Weighting factor.
Pfn	Penalty factor for NO_{x} emission.
a_i, b_i, c_i	Fuel cost coefficients of ith DER.
$\alpha_i, \beta_i, \gamma_i$	Emission coefficients of ith DER.
η_{ith}	Thermal efficiency of ith DER.
η_{ex}	Efficiency of heat exchanger.
H_D	Heat demand (kWh).
LSI	Loss sensitivity index.

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I. Introduction

With rapid escalation in fossil fuel price as well as sharp increase in the capital cost of new central generating plant, there is a focused attention on alternate generating system with higher efficiency of energy use. Under deregulation and restructuring of power system, electricity market becomes highly competitive. Today, micro-grid, due to its major technological and regulatory innovation of its small-scale, on-site CHP-based

DERs, has become enabling to compete with traditional centralized electricity plant. Again, as beneficial for power quality and reliability (PQR) of supply to end-users, micro-grid is going to become an attractive alternate source of power to industry, many utilities, commercial buildings, and many other places [1]–[5].

The new clean air policies and regulations have forced electricity generating plants and power producers, called independent power producer (IPP), to consider the environmental impact of DERs in the operation of micro-grid. Under these circumstances, sharing of demand by DERs is not only governed by the units' capability of minimizing the total fuel cost of system generation, but also their capability of satisfying the emission requirements. All CHP-based DERs are responsible for creating atmospheric pollution with the emissions of $\mathrm{NO_x}$, $\mathrm{SO_x}$, $\mathrm{CO_x}$, etc. $\mathrm{NO_x}$ emissions have been the focus of considerable policy effort due to their direct health effects and indirect contribution to ozone levels. The economic emission power sharing is a multi-objective optimization problem that pursues simultaneous compromise between least cost operation and minimum emission level. The present paper uses price penalty factors approach with $\mathrm{NO_x}$ emission (Pfn), which can convert the above-mentioned multi-objective function to a single objective optimization problem. This paper has considered micro-turbines (Mt) and diesel generators (Dg) as two CHP-based DERs and their $\mathrm{NO_x}$ emission in the study. Dgs involve high combustion temperature that result in high $\mathrm{NO_x}$ production, whereas Mts have much lower $\mathrm{NO_x}$ emission because of their lower combustion temperature [6]–[8].

On economic analysis in the context of optimal types, sizes, and locations of distributed generators (DG) in a distribution network or in a micro-grid, modern soft computing techniques, like genetic algorithms, tabu search, evolutionary programming, DER-CAM, etc., have successfully been applied in many research works. Teng *et al.* [9] proposed a value-based method of selection of optimal types, sizes, and locations of DGs, out of fuel cells, mini gas turbines, and solar PV, after proper codification in genetic algorithms (GA) method. Hernandez-Aramburo *et al.* [10] aimed at developing a unit commitment operation in a micro-grid on optimal fuel consumption with constraints of local heat and electricity demand balance as well as provision for certain minimum reserve power. Authors imposed penalty on excess heat generation and, finally, claimed their solution strongly supports the communication infrastructure. Mitra *et al.* [11] presented a dynamic programming-based analysis on a six-bus meshed micro-grid for finding out optimal mix of DERs among micro-turbines, solar PV (i.e., time varying capacity), and battery storages to meet both electrical and thermal loads. Imposing reliability constraint authors minimized the cost, which consisted of deployment cost, heat compensation cost, and fuel cost. Hatziargyriou *et al.* [12] addressed the unit commitment problem assuming linear continuous and convex bid functions for DG as well as loads along with market price. But the economic dispatches of regulated DGs were handled using monthly 24-hour typical emission curve to incorporate environment impact. Pipattanasomporn *et al.* [13] developed a optimal mix of DG model using mixed-integer linear program with $\mathrm{NO_x}$ emission as one of the constraints. Marnay *et al.* [14] used DER-CAM optimization technique for minimizing cost of combination of equipments, including CHP equipments and renewable sources, for commercial building, and authors reported carbon emission reduction in their results. Distributed Energy Resources Customer Adoption Model (DER-CAM) is a fully technology-neutral optimizing model of economic DER adoption, written in the General Algebraic Modeling System (GAMS) software. Its objective is to minimize the operating cost of on-site generation and CHP systems, for either an individual customer site or a micro-grid. It was developed at Berkeley Laboratory, USA. Hawkes *et al.* [15] developed a linear programming-based unit commitment for a micro-grid with an object to minimization of equivalent annual cost of meeting a given energy (electricity and heat) demand profile. The present paper discusses, briefly, on bus-location and size selection of DERs [16], [17] and tabulates results in the context of a 14-bus radial micro-grid. However, main focus is beamed on the economic emission load dispatch (EELD), both thermal and electric, using differential evolution (DE) algorithms [18], [19]. DE is found to yield better and faster solution, satisfying all the constraints, both for uni-modal as well as multi-modal systems, using its different crossover strategies. It is a simple population-based stochastic parallel search evolutionary algorithm for global optimization. EELD results obtained by DE are verified by PSO [20] and both results are compiled in the tabular form. PSO algorithm is summarized as simple concept, easy implementation, robustness to control parameters, and computationally efficient when compared with other heuristic optimization techniques.

The contents of this paper are organized into eight sections. Following the Nomenclature and Section I, Section II provides detailed formulations of the problem. Section III gives a brief overview of DE technique. Section IV details the DE algorithms in the context of present EELD problem. Section V includes necessary figures, results, and discussions of the study case. The conclusion is drawn in Section VI. References and biographies are appended last.

II. Problem Formulation

The present paper addresses, mainly, the EELD-based scheduling of DERs for proper energy management planning. As DERs siting and sizing are relevant in the present context, so their formulations are added additionally with EELD.

A. Bus-Location Selection of DERs Using LSI

Loss sensitivity (1) based on Newton-Raphson load flow method is used to find out the optimal placement of DERs:

$$\frac{\partial \mathrm{P}_L}{\partial \mathrm{P}_i} = [J_{L1}] * \left[\frac{\partial \mathrm{P}_L}{\partial \delta_i}\right] \tag{1}$$

where $[J_{L1}]$ is Jacobian sub-matrix of $[J^T]^{-1}$, i.e., containing all $[\partial\delta_\mathrm{i}/\partial\mathrm{P_i}]$ terms. P_i is a function of both δ_i and U_i in (2):

$$P_L = \sum_{i=1}^{N_b} P_i(\delta_i, U_i). \tag{2}$$

Change of P_L with change of P_i depends on change of both δ_i and U_i. As U_i change can easily be adjusted, therefore, its effect on P_L change has been neglected here; only δ_i effect has been taken into account. N_b is total number of buses in the network [16], [17].

B. Size Selection of DER

For system loss (P_L) minimization, objective function is given by

$$Min\left(P_L = \sum_{i=1}^{N} PG_i - P_D\right). \quad (3)$$

Optimization is done subject to the PQR constraints as given below.

1) Bus Voltage Tolerance Limit:

$$U_{i\min} \leq U_i \leq U_{i\max}$$

2) Limit on the Active and Reactive Power Generation of the DER:

$$PG_{i\min} \leq PG_i \leq PG_{i\max}$$
$$QG_{i\min} \leq QG_i \leq QG_{i\max}.$$

3) Line Flow Limits: It must be below thermal limits of line and takes care of internal congestion of the micro-grid

$$S_{ij} \leq S_{ij\max}.$$

4) Zero Slack Bus Injection: P_1 and Q_1 are made as small as possible (nearly zero). This reduces the power drawn from utility to zero. Zero slack bus injection constraint helps to know, at the planning stage, what exact DER capacities are required to meet the internal demand of micro-grid.

C. Economic Emission Load Dispatch (EELD)–Both Thermal and Electric

Cost function of EELD is given in (4):

$$Minimize\,(C = W \times f_1 + (1 - W) \times (Pfn) \times f_2). \quad (4)$$

Here, C is the total cost of emission and fuel. Pfn blends NO_x emission cost (f_2) with the normal fuel cost (f_1). NO_x has been considered, presently, as the only one pollutant for analysis. W is the weighting factor whose value varies uniformly between [0, 1].

Above optimization is done subject to following constraints.

1) Power Balance:

$$\sum_{i=1}^{N} PG_i - P_D - P_L = 0. \quad (5)$$

It is common practice to express the network loss P_L as a quadratic function of the generator power outputs through B-coefficients and its general form containing a linear term and a constant term [8], [21], referred to as Kron's loss formula, is

$$P_L = \sum_{i=1}^{N}\sum_{j=1}^{N} PG_i B_{ij} PG_j + \sum_{i=1}^{N} B_{0i} PG_i + B_{00} \quad (6)$$

where $B_{ij}, i, j = 1, \ldots N$, are called the loss-coefficients; their units are MW^{-1}. They can be regrouped to form a symmetrical square matrix of dimension $(N \times N)$. Unit of B_{00} matches that of P_L and it contains a single element, while units of B_{0i} are dimensionless and elements of B_{0i} form $(1 \times N)$ matrix.

Dependent virtual utility generator capacity (PG_1) is related by following (7):

$$B_{11}PG_1^2 + \left(2\sum_{i=2}^{N} B_{1i}PG_i + B_{01} - 1\right)PG_1 + \left(P_D + \sum_{i=2}^{N}\sum_{j=2}^{N} PG_i B_{ij} PG_j + \sum_{i=2}^{N} B_{0i}PG_i - \sum_{i=2}^{N} PG_i + B_{00}\right) = 0. \quad (7)$$

Equation (7) can be simplified as

$$XPG_1^2 + YPG_1 + Z = 0 \quad (8)$$

where

$$X = B_{11} \quad (9)$$
$$Y = 2\sum_{i=2}^{N} B_{1i}PG_1 + B_{01} - 1 \quad (10)$$
$$Z = P_D + \sum_{i=2}^{N}\sum_{j=2}^{N} PG_i B_{ij} PG_j + \sum_{i=2}^{N} B_{0i}PG_i - \sum_{i=2}^{N} PG_i + B_{00}. \quad (11)$$

The real roots of (8) are obtained as

$$PG_1 = \frac{-Y \pm \sqrt{Y^2 - 4XZ}}{2X}, \text{ where } Y^2 - 4XZ \geq 0. \quad (12)$$

To satisfy the equality constraint of (5), the positive root of (12) is chosen as output of the dependent first generator.

2) DER Capacity Limits Constraint: As the power generated by DER shall be within their lower limit $\mathrm{PG_{imin}}$ and upper limit $\mathrm{PG_{imax}}$, so that

$$PG_{i\min} \leq PG_i \leq PG_{i\max}. \quad (13)$$

3) Heat Balance Inequality Constraint: Considering heat output (H_R) of Dg and Mt are proportional to their respective electric output

$$H_R = Total\ heat\ output = \sum_{i=1}^{N} \theta_i PG_i. \quad (14)$$

Heat balance inequality constraint is given as follows:

$$\sum_{i=1}^{N} \theta_i PG_i \geq H_D. \quad (15)$$

θ_i is proportionality constant, called heat-to-power ratio of the ith DER and determined from heat rate using (16). Unit-wise heat exchanger has been considered:

$$\theta_i = \frac{\text{Heat Rate}\left(\frac{kJ}{kWh}\right)}{3600} \times \eta_{ith} \times \eta_{ex}. \quad (16)$$

D. Steps to Find Out Pfn for NO_x [8], [22]

The procedural steps to find out the price penalty factors for $\mathrm{NO_x}$ emissions (Pfn) are as follows.

1) Fuel Cost: The fuel cost of each DER is evaluated at its maximum output in \$/h is

$$f_{1\max} = \sum_{i=1}^{N} \left(a_i + b_i \times PG_{i\max} + c_i PG_{i\max}^2\right). \quad (17)$$

2) NO_x Emission: $\mathrm{NO_x}$ emission release of the ith DER is evaluated at its maximum output in g/kWh as

$$f_{2\max} = \sum_{i=1}^{N} \left(\alpha_i + \beta_i \times PG_{i\max} + \gamma_i \times PG_{i\max}^2\right). \quad (18)$$

Emission coefficients (α_i, β_i, and γ_i) for $\mathrm{NO_x}$ emission of the ith DER are determined applying least squares principle of curve fitting technique on data which are expressed in $\mathrm{NO_x}$ emission versus DER outputs. Similarly, fuel cost coefficients (a_i, b_i, and c_i) are determined from fuel cost versus DER outputs. All such data are obtained from [10] and [23]–[26].

3) $Pfn[i]$: $Pfn[i]$ of the ith DER is calculated as

$$Pfn[i] = \frac{\left(a_i + b_i \times PG_{i\max} + c_i \times PG_{i\max}^2\right)}{\left(\alpha_i + \beta_i \times PG_{i\max} + \gamma_i \times PG_{i\max}^2\right)}. \quad (19)$$

1) Values of $Pfn[i]$ set are arranged in ascending order.
2) Maximum capacity of each unit, $(\mathrm{PG_{imax}})$, is added one at a time, starting from the smallest $Pfn[i]$ unit until

$$\sum PG_{i\max} \geq P_D. \quad (20)$$

3) At this stage, $Pfn[i]$ associated with the last unit in the process is the price penalty factor Pfn for the given load demand.
4) Once the value of Pfn is known, (4) can be minimized subject to the constraints given in (5), (13), and (15).

III. Overview of Differential Evolution Technique

DE is an extremely powerful optimization algorithm from evolutionary computation due to its excellent convergence characteristics and a few control parameters. DE uses a population "IP" of size "NP", at the "gth" iteration, composed of floating point-encoded individuals as per (21), which evolve to reach an optimal solution. Each individual $X_i^{g,}$ of (22) is a vector that contains as many parameters as the problem decision variables D, called "genes". The population size "NP" is a control parameter of the algorithm selected by the user, which remains constant throughout the optimization process:

$$\mathrm{IP^g} = X_i^{\mathrm{g}}, i = 1, \ldots, \mathrm{NP} \quad (21)$$
$$\mathrm{X_i^g} = x_{i,j}^{\mathrm{g}}, j = 1, \ldots, \mathrm{D}. \quad (22)$$

A. Initialization

The optimization process in DE is carried out with three basic operations: mutation, crossover and selection. The first step of this algorithm is to create an initial population of "NP" vectors, by randomly generating individuals within the boundary constraints of (23):

$$\mathrm{IP}^0 = x_{ij}^0 = rand_{i,j} * (H_j - L_j) + L_j \quad (23)$$

where "*rand*" function generates values uniformly in the interval [0, 1]. The fitness function is evaluated for each individual. H_j and L_j are upper and lower limit of boundary constraint of the jth population.

For each generation, the individuals of the population are updated by means of a "Reproduction" scheme. Therefore, for each individual "*ind*", a set of other individuals "π" is randomly extracted.

B. Mutation/Differentiation

The mutation operator is in charge of introducing new parameters into the population. A set of randomly extracted individuals $\pi = \{\xi 1, \xi 2, \ldots, \xi n\}$ is necessary for "Differentiation". To achieve this, mutant operator creates mutant vectors by perturbing a randomly selected vector (ε) with a difference vector δ. The result of "Differentiation", so-called "trial" individual, is

$$\omega = \varepsilon + F * \delta \quad (24)$$

where $F > 0$ is the "constant of differentiation". As for example, three different individuals are randomly extracted from a trial population. The updated trial individual is equal to $\omega = \varepsilon + F{*}\delta$, where $\delta = \xi 2 - \xi 1$ and $\varepsilon = \xi_3$. The scaling constant, F, is an algorithm control parameter used to control the perturbation size in the mutation operator and to improve algorithm convergence. ξ_1, ξ_2, and ξ_3 are randomly chosen vectors and are selected anew for each parent vector.

C. Crossover/Recombination

After the trial, individual "ω" is recombined with updated one "*ind*". Recombination represents a typical case of a "genes" exchange. The trial one inherits genes with some probability. Thus

$$\omega = \begin{cases} \omega j, & \text{if } randj < Cr \\ indj, & \text{otherwise} \end{cases} \quad (25)$$

where $j = 1, \ldots, D$ and $Cr \in [0, 1)$ is the "constant of recombination". Crossover constant C_{r} is an algorithm parameter that controls the diversity of the population and aids the algorithm to escape from local optima.

D. Selection

Selection is realized by comparing the cost function values of updated and trial individuals. If the trial individual has lower value of the cost function, then it replaces the updated one:

$$ind = \begin{cases} \omega, & \text{if } f(\omega) \leq f(ind) \\ ind, & \text{otherwise.} \end{cases} \quad (26)$$

It may be noticed that there are only three control parameters in this algorithm. These are "NP" (population size), "F" (constant of differentiation), and "Cr" (constant of recombination). As for the termination conditions, one can either fix the number of generations "$g_{\max}$" or a desirable precision of a solution. DE offers several variants or strategies for optimization. These can be denoted by DE/x/y/z, where x refers to the vector used to generate mutant vectors, y the number difference vectors used in the mutation process, and z the crossover scheme used in the crossover operation.

IV. DE-Based Algorithms for EELD

Differential evolution can be adjusted to solve the economic emission load dispatch (EELD) problem. Let $p_i = [(PG_{i1}, PG_{i2}, \dots PG_{iN})]$ be the trial vector designating the ith particle of the population and $i = 1, 2, 3 \dots \mathrm{NP}$. The elements of p_i are real power outputs of N generating units. The objective is to minimize the function as mentioned in (4). Set the value of "W" starting from "0". Divide the interval (0, 1) into 40 subintervals. The corresponding DE algorithm can be described by the following steps:

1) Input the system data consisting of fuel cost curve coefficients and emission level coefficients of generators, power generation limits, weighting factor "W", load demand, transmission loss coefficients for that load demand.
2) Initialize the particles of the population in a random manner according to the limits of each unit including individual dimensions, search points, and velocities. These initial particles must be feasible candidate solutions that satisfy the practical operating constraints.
3) Fitness function "C" is evaluated as per (4), after calculating "Pfn" using (19) for each individual set of the population.
4) Apply the Differentiation (Mutation) operation on the population as per (24).
5) Apply the Crossover (Recombination) operation on the population, generated after mutation operation of Step 4), as per (25).
6) The population settings after Steps 4) and 5), which perform better against the fitness function, are selected to be part of the next population according to (26).
7) If the current iteration is greater than or equal to the maximum iteration, keep the result in an Array (known as Pareto-optimal set) and stop; otherwise, repeat Steps 3)–6).
8) Increment the value of "W" in step of 0.025 and repeat the steps starting from Step 1) to Step 7). Repeat this process until the value of "W" reaches to 1.
9) Best Compromise Solution—The algorithm described above generates the non-dominated set of solutions known as the Pareto-optimal solutions. The decision maker (power system operator) may have imprecise fuzzy goal for each objective function. To aid the operator in selecting an operating point from the obtained set of Pareto-optimal solutions, fuzzy logic theory is applied to each objective function to obtain a fuzzy membership function μf_i as per (27):

$$\mu f_i = \begin{cases} 1, & f_i \leq f_{i\,\min} \\ \frac{f_{i\,\max} - f_i}{f_{i\,\max} - f_{i\,\min}}, & f_{i\,\min} < f_i < f_{i\,\max} \\ 0, & f_i \geq f_{i\,\max}. \end{cases} \quad (27)$$

TABLE I
Line Data—14-Bus System

Line no.	Start Bus	End Bus	R (p.u.)	X (p.u.)	B (p.u.)
1	1	2	0.0133	0.042	0.0063
2	2	3	0.0194	0.059	0.026
3	3	4	0.0312	0.16	0.028
4	2	5	0.023	0.12	0.0071
5	5	6	0.023	0.12	0.0071
6	6	7	0.0193	0.059	0.0
7	6	8	0.032	0.084	0.0
8	7	9	0.034	0.17	0.0
9	2	10	0.016	0.042	0.008
10	10	11	0.193	0.059	0.026
11	11	12	0.067	0.17	0.017
12	12	13	0.04	0.1	0.0
13	11	14	0.05	0.15	0.0

TABLE II
Bus Data—14-Bus System

Bus No.	14- Bus system	
	Real load demand (kW)	Reactive load demand (kvar)
1	0.0	0.0
2	20.0	6.0
3	85.0	27.0
4	40.0	1.0
5	20.0	6.0
6	20.0	6.0
7	76.0	16.0
8	10.0	30.0
9	61.0	16.0
10	12.0	75.0
11	10.0	90.0
12	16.0	61.0
13	90.0	59.0
14	35.0	61.0

The best non-dominated solution can be found when (28) is at maximum where the normalized sum of membership function values for all objectives is highest:

$$\mu^k = \frac{\sum_{i=1}^{N} \mu^k fi}{\sum_{k=1}^{M} \sum_{i=1}^{N} \mu^k fi}. \quad (28)$$

In (28), M is the number of non-dominated solutions. After completing the process, best solution of the EELD problem is found.

V. Case Study

This paper conducts study on a 4-DER 14-bus hypothetical radial micro-grid. Line data and bus data of the 14-bus system are shown in Tables I and II, respectively. The system is developed in a similar way as the authors' previous work [16]. Utility as a virtual generator is connected to slack bus 1 and acting as a spinning reserve during the period of analysis.

B-coefficients are dependent on both locations and sizes of the DERs in the network. Sizes are also required to know for multi-objective EELD problem, so that no oversized DER is placed at any bus. Evaluation of sizing of CHP-based DERs using loss minimization is independent of their types. Type is,

TABLE III
DERs Data -Fuel and Emission Coefficients, Operating Limits, Heat Rates

Bus No.	1	2	6	11	12
DER Capacity (kW)	500 (Dg)	200 (Dg)	80 (Mt)	100 (Dg)	30 (Mt)
a_i	10.193	2.035	0.5768	1.1825	0.338
b_i	105.18	60.28	57.783	65.34	89.1476
c_i	62.56	44.0	-133.0915	44.0	-547.619
α_i	26.55	14.4296	3.0358	19.38	1.0346
β_i	-16.1836	-64.1535	-57.3403	-176.6946	-60.384
γ_i	7.0508	130.4094	311.5728	821.6573	943.1898
PG_{imax} (kW)	500	200	80	100	30
PG_{imin} (kW)	0.00	40.0	16.0	20.0	6.0
Heat Rate (kJ/kWh)	10314	11041	11373	10581	12186

again, the main factor for present EELD study. Therefore, for EELD-based energy management planning of a micro-grid, it is relevant to know the optimal sitings and sizings of strategically deployed DERs. However, the main focus of the present work is to study how demands, both electric and thermal, could be shared by DER-mix under EELD condition.

B-coefficients, efficiency of heat exchanger, DE, and PSO data used in the studies are shown below and Table III shows the data of DERs [10], [16], [23]–[27].

1) *B-coefficients*:

$$B_{ij} = \begin{bmatrix} 0.4355 & -0.1694 & 0.1482 & -0.2684 & -0.0925 \\ -0.1694 & 0.2366 & -0.0247 & -0.0061 & -0.0689 \\ 0.1482 & -0.0247 & 0.1636 & -0.2391 & -0.1046 \\ -0.2684 & -0.0061 & -0.2391 & 0.6517 & 0.1987 \\ -0.0925 & -0.0689 & -0.1046 & 0.1987 & 0.1864 \end{bmatrix}$$

$$B_{0i} = [-0.0326 \quad -0.0314 \quad 0.0057 \quad -0.0018 \quad 0.0050]$$

$$B_{00} = [0.0014]$$

2) Efficiency of heat exchanger: 90%
3) PSO data [16], [17]:
Population size: 60; Learning factors: $C_1, C_2 = 2$; Generation or iteration $= 1500$; Inertia weight factor: $w_{\max} = 0.95$ and $w_{\min} = 0.2$. Constriction Factor $= 1$.
4) DE data: using strategy DE/rand/1
Population size $= 60$, Scaling factor, or, constant of Differentiation $(F) = 0.85$, Crossover constant, or, constant of Recombination $(Cr) = 1$.

Following studies are conducted on the test micro-grid:

A. Optimal Siting and Sizing of DERs

Optimal sitings of DERs are selected on the basis of LSI of buses. Fig. 1 plots the LSI versus bus number. All negative LSI values of buses at the fourth quadrant are brought to the first quadrant by shifting the abscissa downward by a suitable dimension. Though terminal bus possesses higher negative LSI value compared to other buses on the same feeder, siting of DER at terminal bus is avoided on the reliability ground. Due to higher outage probability of feeder sections, there are higher chances of under utilization of DER capacity at terminal bus because of islanding from the rest of the network. Compromising between LSI value and reliability, arbitrarily selected four DERs here, have been located at three junction buses 2, 6, and 11 and fourth one at bus 12, which assumes LSI value next higher to terminal bus 13 on the same feeder. Also, at peak demand of 495 kW and without DER, voltage obtained at terminal bus 13, by Newton-Raphson load flow method, is 0.879 p.u., which is the lowest minimum among all 14 network buses.

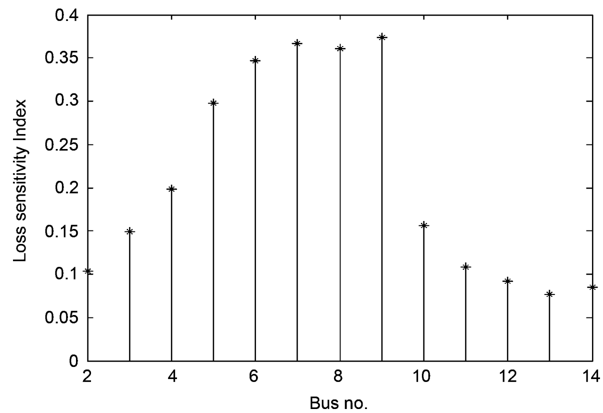

Fig. 1. LSI Plot of 14-bus system.

Optimal sizes of DERs are evaluated at minimum system loss using PSO, and results of simulation obtained at zero slack bus constraint are obtained as 250 kW (at bus 2), 80 kW (at bus 6), 139 kW (at bus 11), and 30 kW (at bus 12).

B. Bi-Objective Optimization

EELD study has been covered in this subsection. To maintain the DERs capacity sizes within the limit as obtained in subsection (A), 200 kW Dg at bus 2, 80 kW Mt at bus 6, 100 kW Dg at bus 11, and 30 kW Mt at bus 12 are selected. A 500 kW or higher capacity Dg is assumed as dependent virtual utility generator covering maximum demand of 495 kW. Data for fuel consumption of Mts and Dgs have been collected from [10] and [24], respectively. Emission data of Mts are obtained from [23] and that of Dgs from [25] and [26]. All these data are curve fitted, interpolated as well as extrapolated by a second-order polynomial to obtain a convex nature between 20% and 100% of rated power of respective DER. Thermal efficiency of all Dgs have been taken as 30% and that of Mts as 50% [16]. As it is an energy management planning of micro-grid, authors try to find out how a particular electric demand could be shared solely by DERs without participation of utility, i.e., at zero slack bus injection. This could be obtained putting comparable weight to fuel cost and emission coefficients of 500 kW Dg (Table III). Characteristics of Mts and Dgs are such that their emissions per hour per unit output (here, in g/kWh) decrease with the increase of each of their kW outputs towards respective rated values, but reverse are the cases for fuel consumption and heat output. Again, from the data of the present study, it is observed that for the same output, $\mathrm{NO_x}$ emissions of both Dgs are several times higher than that of Mts. Also fuel consumption cost is higher for Dgs whereas kWh heat output is lower when compared with

TABLE IV
RESULTS AT OPTIMAL EMISSION ($W = 0$) AND AT OPTIMAL FUEL COST ($W = 1$)

Method	Load (kW)	W	DERs output at Buses (kW) 1	2	6	11	12	Fuel cost ($/hr)	NO_x (g/kWh)	Heat (kWh)
DE	169	0	0.0	63.90	19.40	81.60	8.20	25.750	50.490	157.740
		1	0.0	63.20	80.00	20.30	6.30	23.9689	54.7162	191.716
	248	0	0.0	113.30	35.10	88.30	15.40	30.9433	47.303	237.243
		1	0.0	110.10	80.00	29.10	30.00	29.380	50.950	273.510
	338	0	0.0	166.50	58.30	96.10	21.50	36.851	44.820	329.790
		1	0.0	166.30	80.00	64.30	30.00	35.8974	45.8467	348.048
PSO	169	0	0.0	67.85	17.08	82.11	6.00	25.740	50.490	154.740
		1	0.0	63.86	80.00	20.00	6.00	23.960	54.740	191.487
	248	0	0.0	112.72	36.17	88.48	14.78	30.915	47.300	237.430
		1	0.0	108.70	80.00	30.48	30.00	29.390	50.820	273.490
	338	0	0.0	166.20	58.64	96.07	21.66	36.840	44.820	330.070
		1	0.0	166.68	80.00	63.89	30.00	35.897	45.870	348.000

TABLE V
BEST COMPROMISED SOLUTION BY PARETO

Method	Load (kW)	DERs output at Buses (kW) 1	2	6	11	12	Fuel cost ($/hr)	NO_x (g/kWh)	Heat (kWh)
DE	169	0.0	40.55	75.44	48.28	6.23	24.2400	52.530	188.5500
	248	0.0	87.84	80.00	76.48	6.36	29.5250	48.155	256.7050
	338	0.0	150.54	80.00	90.92	20.55	36.0720	45.020	341.7225
PSO	169	0.0	40.00	73.57	51.11	6.00	24.3028	52.330	187.2958
	248	0.0	89.25	80.00	75.42	6.00	29.5160	48.170	256.4900
	338	0.0	150.20	80.00	89.86	21.95	36.0600	45.030	342.7200

Mts at same kW output. Results (Tables IV and V) of the study reveal following valuable information, which conform to their characteristics and help in energy management planning of the micro-grid:

1) At lower electric demand tracking, i.e., 169 kW, range of heat output is wide i.e., from minimum value of 157.74 kWh at optimal emission condition ($W = 0$) to maximum value 191.76 kWh at optimal fuel cost ($W = 1$) (Table IV). Heat demand within this range could be served by DER-mix simultaneously with particular electric demand of 169 kW. If the electric demand to be tracked increases, corresponding range of heat output is narrowed down. If heat demand exceeds the range, alternative source, like back-up boiler, is to be installed.
2) Like heat demand, fuel cost as well as emission ranges are narrowing down with increase of demand. At higher demands, all DERs approach towards their respective maximum capacity limit and thus chances of shuffling their outputs get narrowed.
3) As utility, i.e., 500-kW virtual generator, acts as a spinning reserve, its a_1 and α_1 coefficients help set up reserve charge to be imposed on the owner.
4) Table V shows the Pareto optimal results. Comparing with results of Table IV, it is noticed that there is a compromization between fuel cost and emission.

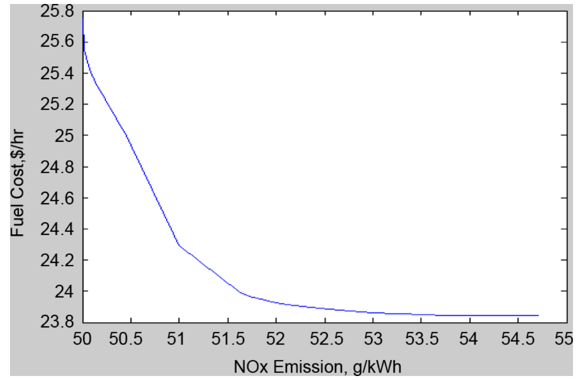

Fig. 2. Pareto optimal front for fuel cost and NO_x emission at 169-kW electric demand with DE.

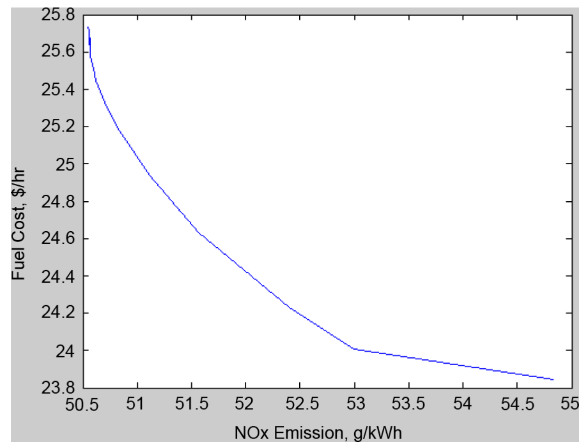

Fig. 3. Pareto optimal front for fuel cost and NO_x emission at 169-kW electric demand with PSO.

5) Figs. 2 and 3 depict the Pareto optimal front for fuel costs and NO_x emissions at 169-kW electric demand obtained using DE and PSO, respectively.
6) Figs. 4 and 5 are the 3-D plot of optimal front showing the relations among fuel costs, NO_x emissions, and heat demands at 169-kW electric demand tracking with DE and PSO, respectively.
7) Fig. 6 depicts the change of NO_x optimization with waste heat utilization at 169-kW electric demand. With the increase of heat output at same demand, NO_x emission increases due to shift of generation from diesel to micro-turbine. Similar trends are observed at other two electric demands, but range of heat output is shrunk at higher electric demand.
8) At constant heat demand, optimal emission and optimal fuel cost are, respectively, 0.14% and 0.4% sensitive to per kW changing load at 169 kW. Almost similar sensitiveness is achieved at other two demands.

C. Comparison Between (DE) and (PSO)

Results obtained by both simulation techniques are tallying each other (Tables IV and V). The only difference is that DE algorithm is faster than PSO (Table VI). The program is written and run in MATLAB 7 using Pentium-4 PC with 512 MB of RAM. For 1500 iterations with 60 population size, minimum and maximum time elapsed using DE are 115.97 s and

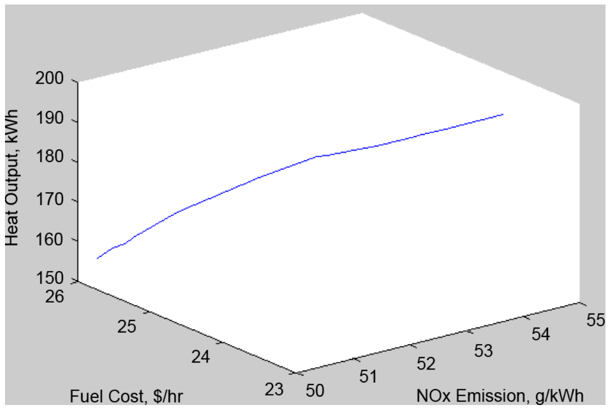

Fig. 4. 3-D plot of optimal front showing the relations among fuel cost, NO_x emission, and heat demand at 169-kW electric demand tracking with DE.

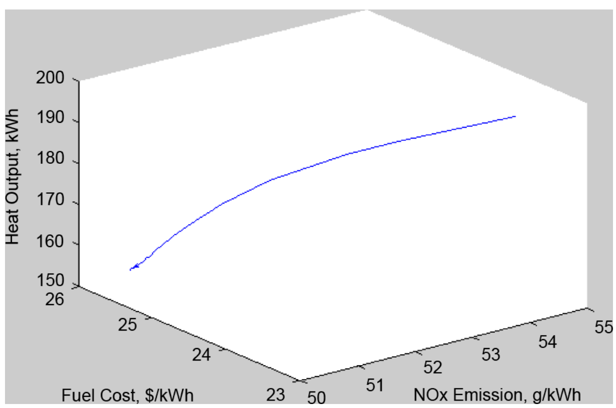

Fig. 5. 3-D plot of optimal front showing the relations among fuel cost, NO_x emission, and heat demand at 169-kW electric demand tracking with PSO.

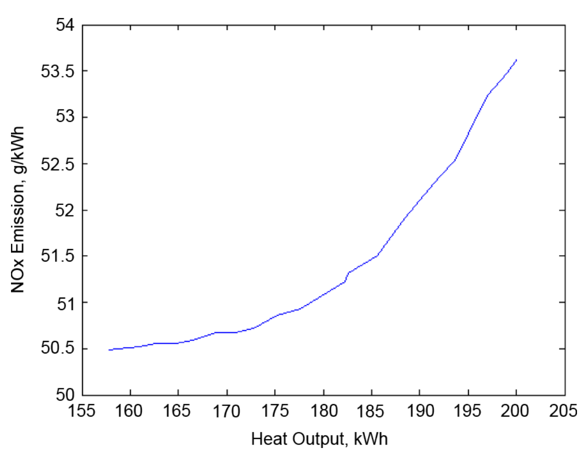

Fig. 6. Heat output versus NO_x emission at 169-kW electric demand.

TABLE VI
COMPARISON OF TIME (SECONDS)

DE, 1500 iterations & average for 40 trials			PSO, 1500 iterations & average for 40 trials		
Min.	Max.	Average	Min.	Max.	Average
115.97	143.63	125.609	134.078	320.625	249.016

143.63 s, respectively, whereas those with PSO are 134.078 s and 320.625 s.

VI. CONCLUSION

Both air pollution and fuel shortage are the burning issues with which all the world is concerned. As a result of it, every country is striving to shift from its conventional fossil fuel-based generating system to one like micro-grid. Both emission and fuel costs are related to O&M cost of DERs. Energy management of micro-grid is largely dependent on both fuel cost and emission, which, in turn, helps make the micro-grid competitive in deregulated market. In the context of a 14-bus radial micro-grid, the present paper proposes an original idea to incorporate in the optimization technique by which owners could make a schedule to cater a particular electric demand and its corresponding range of heat demands solely using the DER-mix at different weight of compromisation between fuel cost and emission. This method shows one of the many avenues of economical analysis. There are a number of other factors, such as type of manufacturer and technology of DERs on which both fuel consumption and emission depend. Again, policies of local utility, as well as government regarding emission, affect the analysis. Results obtained, independently, by DE as well as PSO techniques confirm what economical mix of DERs would be in operation to cater different loads and corresponding heat demands. Future study can be extended with use of other techniques, systems, and renewable sources.

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